

# Controlled Soft Breaking of N=1 SQCD

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## Abstract

We discuss the introduction of soft breaking terms into the exact solutions of N=1 SQCD using a spurion analysis. The spurion symmetries are not sufficient to determine the behavior of models in which squark or gaugino masses alone are introduced. However, a controlled approximation is obtained in some cases if a supersymmetric mass is first introduced for the matter fields. We present low-energy solutions for two models with perturbing soft breaking terms, one with a gaugino mass and one with squark mixing. These models have non-trivial theta angle dependence and exhibit phase transitions at non-zero theta angle analogous to those found in the chiral Lagrangian description of QCD.

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# 1 Introduction

The exact solutions [1, 2, 3] for the IR Wilsonian effective theory of  $N=1$  supersymmetric QCD (SQCD) reveal some surprising dynamical effects. Most striking are the occurrence of massless composite bound states (or solitons) in the strong coupling regime. It is intriguing whether these massless states could smoothly map to states important to the dynamics of non-supersymmetric gauge theories. It is highly implausible that the massless composite fermions of SQCD can survive in the QCD limit. The lattice arguments of Weingarten [4] imply that any composite states in QCD must be heavier than the pions. Nevertheless, it is possible, for example, that the scalar “dual quark” solitons might survive in some form and be involved in some “dual magnetic” description of confinement in QCD.

Soft breaking terms, such as squark and gaugino masses, may be introduced to the SQCD theories as spurion fields with non-zero F-component vevs that explicitly break supersymmetry [5]. The symmetries of the enlarged spurion model constrain how they may appear in the low energy Wilsonian theory. In general these constraints are not however sufficient to determine the low energy theory since “Kahler Potential” terms may be constructed that are invariant to all symmetries and are hence unknown [6]. For the cases where a squark and/or gaugino mass are the sole supersymmetry and chiral symmetry breaking parameters these Kahler terms dominate the behaviour of the potential. Some speculations as to the behaviour of these theories were made in Refs [8, 9].

In this paper we discuss these difficulties and investigate some cases in which the effects of the soft breakings *can* be controlled. We start with a model with supersymmetry preserving quark/squark masses, and then break supersymmetry with squark and gaugino masses resulting from spurions that occur linearly in the superpotential. It can be shown that any possible Kahler corrections are higher order in the soft breakings, and thus control may be retained over the low-energy theory. The analysis is similar to that performed on the  $N=2$  SQCD solutions in Ref [6]. The derivative (low-energy) expansion performed to obtain the solutions of SQCD restricts the solutions of the softly broken models to the regime where the soft breakings are small relative to the strong interaction scale. At first sight the resulting models appear to behave almost identically to their supersymmetric counterparts but, as for the  $N=2$  solutions [7], the models have the additional new feature of displaying  $\theta$  angle dependence. The softly broken models distinguish the  $N_c$  vacua of the SQCD models and as  $\theta$  is changed these vacua interchange at first order phase transitions. We contrast this behaviour with that of the QCD chiral Lagrangian.

## 2 N=1 SQCD

We begin from the N=1  $SU(N_c)$  SQCD theories with  $N_f$  flavors described by the UV Lagrangian

$$\mathcal{L} = K^\dagger K (Q_i^\dagger Q_i + \tilde{Q}_i^\dagger \tilde{Q}_i)|_D + \frac{1}{8\pi} Im \tau W^\alpha W_\alpha|_F + 2Re m_{ij} Q_i \tilde{Q}_j|_F \quad (1)$$

where  $Q$  and  $\tilde{Q}$  are the standard chiral matter superfields and  $W^\alpha$  the gauge superfield. The coupling  $K$  determines the kinetic normalization of the matter fields. The gauge coupling  $\tau = \theta/2\pi + i4\pi/g^2$  defines a dynamical scale of SQCD:  $\Lambda^{b_0} = \mu^{b_0} exp(2\pi i \tau)$ , with  $b_0 = 3N_c - N_f$  the one loop coefficient of the SQCD  $\beta$ -function. And, finally,  $m$  is a supersymmetric mass term for the matter fields. We may raise these couplings to the status of spurion chiral superfields which are then frozen with scalar component vevs. The SQCD theory without a mass term has the symmetries

|             | $SU(N_f)$ | $SU(N_f)$   | $U(1)_B$ | $U(1)_R$                |
|-------------|-----------|-------------|----------|-------------------------|
| $Q$         | $N_f$     | 1           | 1        | $\frac{N_f - N_c}{N_f}$ |
| $\tilde{Q}$ | 1         | $\bar{N}_f$ | -1       | $\frac{N_f - N_c}{N_f}$ |
| $W^\alpha$  | 1         | 1           | 0        | 1                       |

(2)

The mass term breaks the chiral symmetries to the vector symmetry. The classical  $U(1)_A$  symmetry on the matter fields is anomalous and, if there is a massless quark, may be used to rotate away the theta angle. In the massive theory the flavor symmetries may be used to rotate  $m_{ij}$  to diagonal form and the anomalous  $U(1)_R$  symmetry under which the  $Q$ s have charge +1 may be used to rotate  $\theta$  on to the massless gaugino. Including the spurion fields the non-anomalous  $U(1)_R$  symmetry charges are

|     |                         |                         |        |                    |           |
|-----|-------------------------|-------------------------|--------|--------------------|-----------|
| $W$ | $Q$                     | $\tilde{Q}$             | $\tau$ | $m$                | $K$       |
| 1   | $\frac{N_f - N_c}{N_f}$ | $\frac{N_f - N_c}{N_f}$ | 0      | $\frac{2N_c}{N_f}$ | arbitrary |

(3)

The anomalous symmetries may be restored to the status of symmetries of the model if we also allow the spurions to transform. The appropriate charges are

|          | $W$ | $Q$ | $\tilde{Q}$ | $\Lambda^{b_0}$ | $m$ | $K$       |
|----------|-----|-----|-------------|-----------------|-----|-----------|
| $U(1)_R$ | 1   | 0   | 0           | $2(N_c - N_f)$  | 2   | arbitrary |
| $U(1)_A$ | 0   | 1   | 1           | $2N_f$          | -2  | arbitrary |

(4)

The  $m_{ij}$  spurions also transform under the chiral flavor group.

The solutions of the models are  $N_f$  dependent. For  $N_f < N_c$  the low energy superpotential is exactly determined by the symmetries and the theory has a run away vacuum [1]. For

$N_f = N_c$  the low energy theory is in terms of meson and baryon fields

$$\begin{aligned} M_{ij} &= Q_i \tilde{Q}_j \\ b^{[i_1, \dots, i_{N_c}]} &= Q^{i_1} \dots Q^{i_{N_c}} \\ \tilde{b}^{[i_1, \dots, i_{N_c}]} &= \tilde{Q}^{i_1} \dots \tilde{Q}^{i_{N_c}} \end{aligned} \tag{5}$$

subject to the constraint  $\det M + b\tilde{b} = \Lambda^{2N_f}$  [2]. For  $N_f = N_c + 1$  the theory is again described by baryon and meson fields with the classical moduli space unchanged [2].

When  $N_c + 1 < N_f < 3N_c$  the theory has an alternative description of the low energy physics in terms of a dual magnetic theory with an  $SU(N_f - N_c)$  gauge group,  $N_f$  flavors of dual quarks,  $q$  and  $\tilde{q}$ , and  $N_f^2$  meson fields,  $M_{ij}$  [3]. The dual theory has the additional superpotential term  $M_{ij} q_i \tilde{q}_j$ . Generally one of the two duals is strongly coupled whilst the other is weakly coupled (the electric theory is weakly coupled for  $N_f \sim 3N_c$ , the magnetic theory when  $N_f \sim N_f + 2$ ). In the strongly coupled variables the low energy Wilsonian effective theory is a complicated theory with all higher dimensional terms in the superfields equally important (since the IR theory is in a conformal regime the scale  $\Lambda$  at which the theory entered the conformal regime is not available to suppress higher dimension terms and similarly the gauge coupling is order one and may not suppress these operators). The weakly interacting theory however, has a very simple Wilsonian effective theory of the canonical bare form. According to the duality conjecture these two effective theories must describe the same physics and therefore there is presumably a (complicated) mapping between the electric and magnetic variables in the IR.

### 3 Soft Supersymmetry Breaking

Soft breaking interactions terms which explicitly break supersymmetry may be included in the UV theory by allowing the spurions to acquire non-zero  $F$ -components. (These are the terms that can be induced by spontaneous supersymmetry breaking and hence may be included perturbatively while inducing only logarithmic divergences in the theory as a remnant of the supersymmetric non-renormalization theorems [5]). We will consider three such breaking terms, a squark mass ( $F_K \neq 0$ ), a gaugino mass ( $F_\tau \neq 0$ ) and a squark mass mixing ( $F_m \neq 0$ ).

The dependence of the IR effective theory on the spurion fields is determined in the  $N=1$  limit by the dependence on their scalar components, the couplings and masses. The exact solutions of Seiberg, however, do not provide sufficient information to take the soft breakings to infinity limit and obtain results for models with completely decoupled superpartners

since the solutions are only low energy derivative expansions. Higher dimension terms are suppressed by the strong coupling scale  $\Lambda$  and hence in the non-supersymmetric theories there are unknown soft breaking terms of higher order in  $F_S/\Lambda^2$ .

A second problem is that squark masses are only generated through the Kahler potential (the spurion  $F_m$  generates a squark mass mixing but it is unbounded without additional contributions to the masses from the Kahler sector) via such terms as  $|F_S|^2|Q|^2$  with  $S$  a general spurion. There are no symmetry constraints on these terms so we do not know whether they occur in the low energy theory or if they do, their sign. We note that the sign of these terms relative to the sign of the equivalent terms in the UV theory is crucial. As a particular example consider theories close to  $N_f = 3N_c$  where the electric theory has a very weak IR fixed point and the magnetic theory a strongly coupled IR fixed point. We are interested in what happens when we introduce squark and gaugino masses in the UV magnetic theory. We can consider the case where these soft breakings are small relative to the scale  $\Lambda$  at which the theory enters its strongly interacting conformal phase. We expect a conformal phase down to the soft breaking scale but can we say anything about the theory below that scale? The dual squarks in the weakly coupled IR description only acquire masses from  $F_\tau$  and  $F_K$  from the Kahler terms. For infinitesimal soft breakings we do not expect the weakly coupled nature of the dual theory at the breaking scale to be disturbed. If these masses are positive (as investigated in Ref[8]) then below the soft breaking scale the theory behaves like QCD and presumably confines and breaks chiral symmetries at an exponentially small scale relative to the soft breaking masses. Alternatively if the masses are negative (as investigated in Ref[9]) then the magnetic gauge group is higgsed with the possible interpretation in the electric variables of a dual Meissner type effect. The spurion symmetry arguments are not sufficient to distinguish between these possibilities.

It should be remarked that there *is* a strongly coupled magnetic theory that corresponds to the introduction of any soft breaking terms in the electric theory. This is true since we can use the mapping of electric to magnetic field variables from the SQCD theory (which is not known explicitly, but exists in principle) to write the soft breaking terms of the simple weakly interacting theory in terms of the strongly interacting variables in the IR. The result will be a complicated mess of relevant higher dimension operators in the strongly interacting theory. The subtlety is that if we now run the renormalization group back to the UV in the magnetic variables we will, very likely, never recover a weakly interacting theory. At each step to recover the effective theory at the lower scale we must add important higher dimension terms. The problem is therefore to identify which soft breaking terms in the IR electric description correspond to canonical soft breaking terms in the UV magnetic theory.

In the next section we shall resolve this problem for the  $F_\tau$  and  $F_m$  cases after including

a supersymmetric mass that determines the squark masses at order  $F^0$ . Then for small soft breakings relative to  $m$  (and  $\Lambda$ ) exact solutions may be obtained.

## 4 Controlled N=0 Theories

To obtain solutions to softly broken N=1 SQCD theories, we begin by including a supersymmetric mass for the matter fields. The resulting theories have a mass gap on the scale  $m$  and the induced meson  $M_{ij} = Q^i \tilde{Q}_j$  vev is determined independently of  $N_f$  by holomorphy

$$M_{ij} = \Lambda^{\frac{3N_c - N_f}{N_c}} (det m)^{1/N_c} \left( \frac{1}{m} \right)_{ij} = |M_{ij}| e^{i\alpha} \quad . \quad (6)$$

The resulting supersymmetric theories have  $N_c$  distinct vacua corresponding to the  $N_c$ th roots of unity,  $\alpha = 2n\pi/N_c$  (as predicted by the Witten index). Note that for the theories with magnetic duals putting masses in for all flavors breaks the dual gauge group completely. For simplicity henceforth we shall take  $m_{ij}$  to be proportional to the identity matrix; in this basis  $\langle M_{ij} \rangle$  is also proportional to the identity matrix.

These massive theories may be softly broken in a controlled fashion. If the spurion generating the soft breaking enters the superpotential linearly then we may obtain desirable results when that spurion's F-component  $F \ll m \ll \Lambda$ . Any D-term contributions to the scalar potential take the form  $F_X^\dagger F_Y$  with  $X$  and  $Y$  standing for generic fields or spurions. In the supersymmetric limit all F-components are zero and will grow as the vacuum expectation value of the soft breaking spurion. These Kahler terms are therefore higher order in the soft breaking parameter than the linear term from the superpotential. The unknown corrections to the squark masses in the theory are subleading to the masses generated by the supersymmetric mass term and hence we may determine the potential minima at lowest order.

### 4.1 Squark Mass Mixing

The first model we consider includes the bare squark mixing term

$$Re(F_{m\,ij} Q_i \tilde{Q}_j) \quad (7)$$

which is generated from the superpotential. Again for simplicity we will take  $F_{m\,ij}$  to be diagonal with degenerate eigenvalues in the basis in which  $m_{ij}$  is diagonal. The form of the effective theory is governed by the symmetries in (3) which determine that the superpotential of the theory is not renormalized. The soft breaking term is therefore also not renormalized

and is the leading term in an expansion in  $m/\Lambda$ . For  $F_m \ll m \ll \Lambda$  we find that there are the  $N_c$  minima of the SQCD theory given by the values of  $M_{ij}$  in (6) and distinguished by their contribution to the potential

$$\begin{aligned} -\text{ReTr}[F_m M_{ij}] &= -N_f |F_m| |M| \cos([\theta_0 + (N_f - N_c)\theta_m + N_c\theta_f + 2n\pi]/N_c) \\ &= -N_f |F_m| |M| \cos([\theta_{phys} + 2n\pi]/N_c) \quad . \end{aligned} \quad (8)$$

Freezing the spurion  $F_m$  explicitly breaks  $U(1)_R$  and introduces dependence on the  $\theta$  angle.  $\theta_{phys}$  is the correct combination of phases on  $m$ ,  $F_m$  and the bare  $\theta$  angle. To see this in the bare Lagrangian we may use the anomalous  $U(1)_A$  symmetry to rotate any phases on  $F_m$  onto  $m$  and into the  $\theta F\tilde{F}$  term. Then using the anomalous  $U(1)_R$  symmetry under which  $Q_i$  transforms with charge 0 we may rotate the resulting phase on  $m$  into the  $\theta$  angle as well. The resulting  $\theta$  angle is the physical  $\theta$  angle in which the physics is  $2\pi$  periodic:

$$\theta_{phys} = \theta_0 + (N_f - N_c)\theta_m + N_c\theta_f \quad (9)$$

We can also understand the form of (9) as follows. Once the  $U(1)_R$  symmetry is explicitly broken by  $f_m$  a gaugino mass is generated by radiative effects. We can think of  $\theta_{phys}$  as generated by the effective phases on the quark and gaugino masses. The gaugino mass is generated by a perturbative graph with a quark-squark loop. The result is of the form  $F_m/m$ , leading to an effective phase which is  $\theta_f - \theta_m$ . The effective gaugino phase then appears in (9) with an anomaly factor from  $C_2(R)$  of  $N_c$  rather than  $N_f$ . The equivalent effective superpotential term is of the form

$$\ln[m] \text{ } WW \text{ } |_F, \quad (10)$$

which yields another contribution to the potential when the gauginos condense. Using the Konishi anomaly [10], one can see that this term has the same form as (8).

The resulting potential (8) distinguishes the  $N_c$  vacua. For  $\theta_{phys} = 0$  the  $n = 0$  vacua is the true minima.

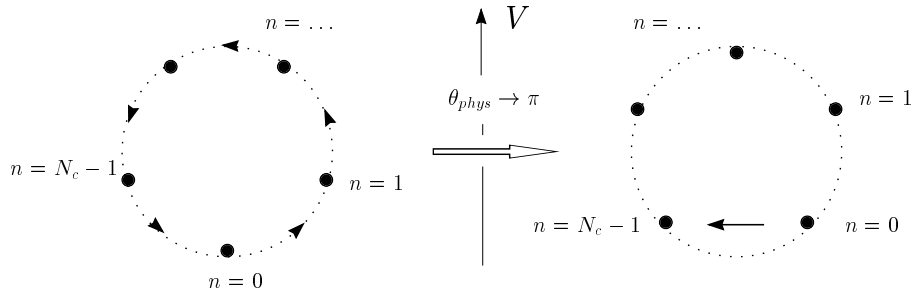


Fig.1: First order phase transition as  $\theta_{phys}$  is varied from 0 to  $\pi$ .

As  $\theta_{phys}$  passes through  $\pi$  the  $n = 0, N_c - 1$  vacua become degenerate and there is a first order phase transition. Then as  $\theta_{phys}$  moves through  $(\text{odd})\pi$  there are subsequent first order phase transitions at which the SQCD minima interchange.

## 4.2 Gaugino Mass

In the UV theory we may induce a gaugino mass through a non zero F-component of the gauge coupling  $\tau$

$$\frac{1}{8\pi} \text{Im}[F_\tau \lambda \lambda] \quad (11)$$

In the IR theory  $\tau$  enters through the strong interaction scale  $\Lambda$  which again occurs linearly in the superpotential of the theory. Taking  $F_\tau \ll m \ll \Lambda$  we again may determine the vacuum structure. The IR superpotential terms compatible with the symmetries of the theory involving  $\Lambda$  are

$$\text{Re}[m M_{ij} + (\det M_{ij})^{1/(N_f - N_c)} \Lambda^{(3N_c - N_f)/(N_c - N_f)}] \quad (12)$$

where the final term results from non-perturbative effects in the broken gauge group. At lowest order in perturbation theory the vev of  $M_{ij}$  is given by (6) which also contains  $\Lambda$  and hence has a non-zero F-component. Including  $F_\tau$  and performing the superspace integral we obtain up to a coefficient the following corrections to the potential that break the degeneracy between the  $N_c$  SQCD vacua

$$\begin{aligned} \Delta V &= -\text{Re} \left[ m^{N_f/N_c} i F_\tau \Lambda^{(3N_c - N_f)/N_c} \right] \\ &= - \left| m^{N_f/N_c} F_\tau \Lambda^{(3N_c - N_f)/N_c} \right| \cos[ \theta_{phys}/N_c + \alpha ] \end{aligned} \quad (13)$$

where again  $\alpha$  are the  $N_c$ th roots of unity and  $\theta_{phys}$  is the physical theta angle in which the physics must be  $2\pi$  periodic. It may be obtained by again making rotations with the anomalous  $U(1)_A$  and  $U(1)_R$  symmetries

$$\theta_{phys} = \theta_0 + N_c(\theta_{F_\tau} + \pi/2) + N_f \theta_m \quad (14)$$

The factor of  $\pi/2$  occurs as a result of the discrepancy between the phase of  $F_\tau$  and that of the canonical definition of the gaugino mass. There is also an additional contribution to the vacuum energy arising from the gaugino condensate. Using the Konishi anomaly [10], we see that it has the same form as (13). The supersymmetry breaking contributions again break the degeneracy between the  $N_c$  supersymmetric vacua. There are again phase transitions as  $\theta_{phys}$  is varied, occurring at  $\theta_{phys} = (\text{odd})\pi$ .



## 5 Discussion

We have investigated some examples where controlled, low-energy descriptions of softly broken massive SQCD may be obtained, despite the lack of supersymmetry. The models we studied are obtained by the inclusion of soft breaking masses from spurions occurring linearly in the superpotential. Examples of such soft breaking terms are gaugino masses and squark mass mixings. The soft breaking corrections to the potential distinguish between the  $N_c$  vacua of SQCD at a generic value of theta angle. At the special values of  $\theta_{phys} = (\text{odd})\pi$  there are first order phase transitions at which two of the  $N_c$  vacua interchange.

This behavior can be compared with the theta angle dependence of the QCD chiral Lagrangian [11] for which there are  $N_f$  distinct vacua which interchange through first order phase transitions at  $\theta = (\text{odd})\pi$ . This difference in the number of vacua between the softly broken theories and QCD would prohibit us from seeing any sign of a smooth transition between the two theories (one might hope that the  $M_{ij}$  vev might smoothly map to the quark condensates of QCD for example) even if we were able to begin to take the squark and gaugino masses towards infinity. There is however one conclusion for QCD that we can tentatively draw from this analysis. In these models the form of the confined effective theory changes smoothly with the theta angle and there is no sign of a break down of confinement as suggested in [12]. This lends some support to the assumption [11] that the chiral Lagrangian remains the correct description of QCD in the IR even at non-zero theta.

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## References

- [1] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. B 241 (1984) 493; Nucl. Phys. B 256 (1985) 557.
- [2] N. Seiberg, hep-th/9402044, Phys. Rev. D 49 (1994) 6857
- [3] N. Seiberg, hep-th/9411149, Nucl. Phys. B 435 (1995) 129.
- [4] D. Weingarten, Phys. Rev. D 51 (1983) 1830.
- [5] N. Evans, S.D.H. Hsu and M. Schwetz, hep-th/9503186, Phys. Lett. B 355 (1995) 475; L. Álvarez-Gaumé, J. Distler, C. Kounnas and M. Mariño, Int. J. Mod. Phys. A11 (1996) 4745.
- [6] N. Evans, S.D.H. Hsu, M. Schwetz and S. Selipsky, hep-th/9508002, Nucl. Phys. B 456 (1995) 205.
- [7] N. Evans, S.D.H. Hsu and M. Schwetz, hep-th/9608135, Nucl. Phys. B 484 (1997) 124. K. Konishi, hep-th/9609021.
- [8] O. Aharony, J. Sonnenschein, M. E. Peskin, S. Yankielowicz, hep-th/9507013, Phys. Rev. D 52 (1995) 6157.
- [9] E. D'Hoker, Y. Mimura, N. Sakai, hep-ph/9611458.
- [10] K. Konishi, Phys. Lett. B 135 (1984) 439.
- [11] E. Witten Ann. Phys. **128** 363 (1980); M.C. Huang and P. Sikivie, Phys. Rev. D32 (1985) 1560; M. Creutz Phys. Rev. D 52 (1995) 2951; N. Evans, S.D.H. Hsu and M. Schwetz, hep-th/9608135.
- [12] G. Schierholz, Nucl. Phys. **B**, Proc. Suppl. 42, 270 (1995).